Exact Spectra Analysis of Sampled Signals With Jitter-Induced Nonuniformly Holding Effects

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***Abstract*—The timing-mismatch/jitter effects in time-interleaved (TI) systems have been traditionally characterized by digital spectra analysis of nonuniformly sampled signals; however, the nature of that output with sample-and-hold (SH) effects has not yet been well established. This paper will present a complete analysis of signal spectra for general TI systems with practical nonuniformly holding outputs imposed by timing-jitter effects. The analysis reveals first the extra spectra representation for different forms of nonuniformly holding signals and then their closed-form expressions of signal-to-noise-and-distortion-ratio (SINAD), in terms of the number of channels, signal frequency, and jitter errors. Such analysis describes both the timing mismatches imposed by random clock-jitter and fixed periodic clock-skew. MATLAB simulation results confirm the accuracy of the derived formula. Finally, such timing effects and the effectiveness of the formula will be demonstrated through the comparison of calculated and simulated results, as well as in some examples of measured data also, obtained from several practical integrated circuit (IC) applications, including a very high-speed data converter, analog front-end (AFE) filtering and an IF sampled-data radio filter.**

***Index Terms*—Clock jitter, nonuniformly holding, nonuniformly sampled-and-held, nonuniformly sampling, time-interleaved (TI), N-path filtering, timing mismatches, timing skew.**

I. Introduction

THE rapid evolution of electronic instruments and data communication demands high-speed data acquisition and conversion channels as well as signal processing units. Time-interleaved (TI) architecture is one of the most effective ways to boost the maximum speed of the analog electronic devices in current process technology e.g., TI analog-to-digital converters (ADCs), digital-to-analog-converters (DACs) [1], [2] and N-path sampled-data filters [3]–[6]. However, the timing-mismatch of clock phases among different TI paths can greatly degrade the system performance [7]–[15], especially for high-speed operation. Such timing mismatches are generally characterized by either a random clock jitter (imposing an increased noise floor over all frequencies) [7], [12], or by an unmatched but fixed periodic timing-skew/offset among different channels (leading to the appearance of image sidebands at multiples of f_s/M, with $f_s$sampling frequency and M-period of timing-skew) [7], [13]–[15], [18]–[21]. Such in-band images cause significant degradation in the dynamic range of system.

Due to the inherent different nature of signal sampling and processing in the applications, such timing-mismatch effects can be represented by the following 3 different processes:

1) If the **I**nput signal is sampled by the system with **N**onuniform time-interval and later played out or represented by discrete samples in the **O**utput at **U**niform time instants for later processing, it can be designated by an **IN-OU** process. This is a typical sampling process in the analog to digital conversion path (timing-mismatch only at input signal sampling) of TI ADCs [1] or multirate sampled-data decimators [4], as shown in Fig. 1(a) with the correspondent signal waveform.
2) If the **I**nput signal is sampled by the system with **U**niformly spaced time-intervals and later the samples are played out in the **O**utput **N**onuniformly, then the system is referred to as **IU-ON** process, that is the typical case in digital to analog conversion path (timing-mismatch only at output signal holding) of TI DACs [2] or multirate sampled-data interpolators [5], as shown in Fig. 1(b).
3) If the **I**nput signal is sampled by the system with **N**onuniform time-interval and then played out **C**orrelatively at the **O**utput with the same **N**onuniform time instants (occurred at the input sampling), such system is named **IN-CON** process. It can typically be applied to a complete TI sampled-data system (timing-mismatches at both input sampling and output holding driven by the same clock phases), as illustrated in Fig. 1(c), e.g., N-path filtering [3].

The signal spectra for these 3 processes with impulse-sampled (IS) sequence form have been well analyzed in [7], [9], [11] respectively. However, in practice, the real output signals are always in sample-and-hold (SH) or holding nature in the latter two processes, as represented in the waveforms of Fig. 1. Fig. 2 presents the plots of FFT output spectra of all IN-OU, IU-ON and IN-CON process with both IS and SH output for a sinusoidal input with normalized frequency $a = f_o/f_s = 0.2$, timing-skew period $M = 8$ and standard deviation of timing-skew ratio $\sigma_{r_m} = 0.1\%$. Unlike the case of IN-OU process, the output signal spectra of the latter two processes are not simply the $\sin(x)/x$ shaped version of the corresponding original impulse-sampled output due to the nonuniformly holding effect, e.g., compare the calculated

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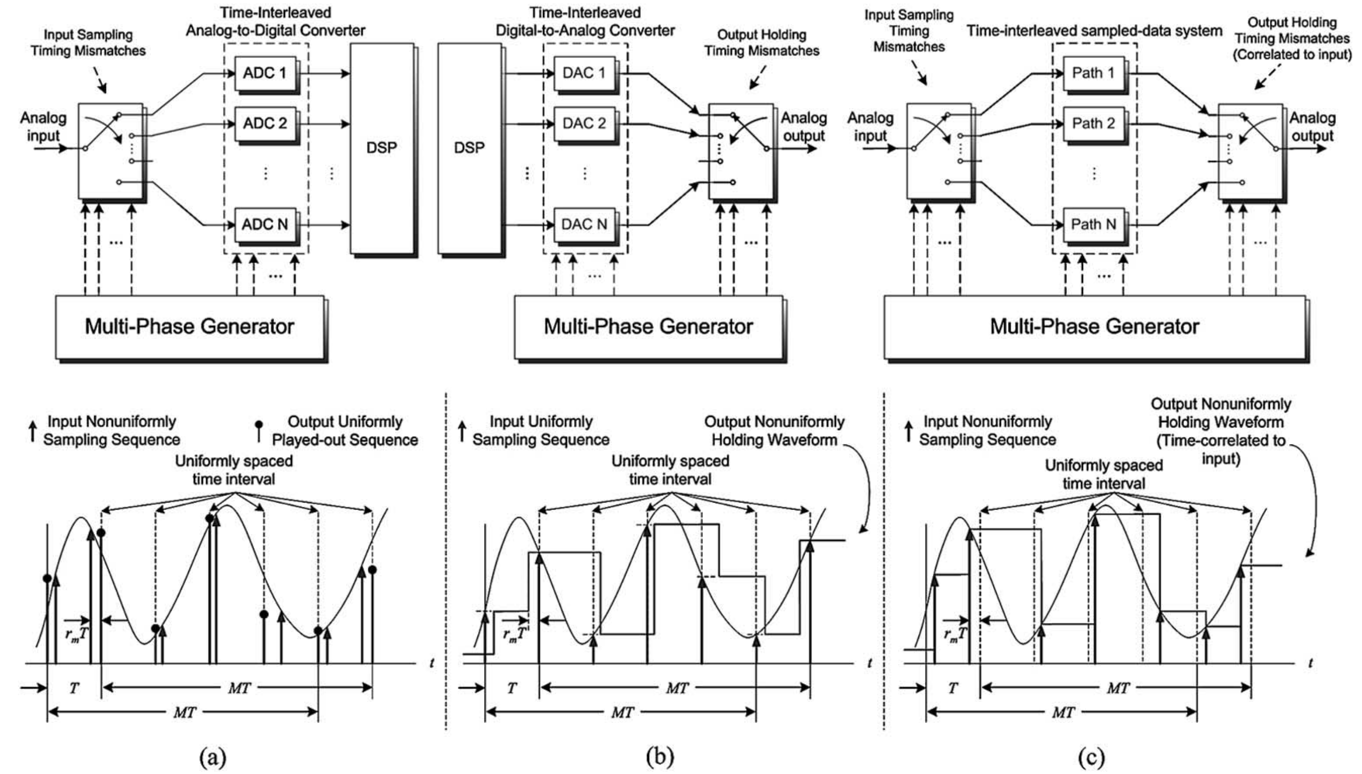

Fig. 1. Equivalent (a) IN-OU(IS) (b) IU-ON(SH) (c) IN-CON(SH) processes for TI ADC, DAC, and Sampled-data Systems.

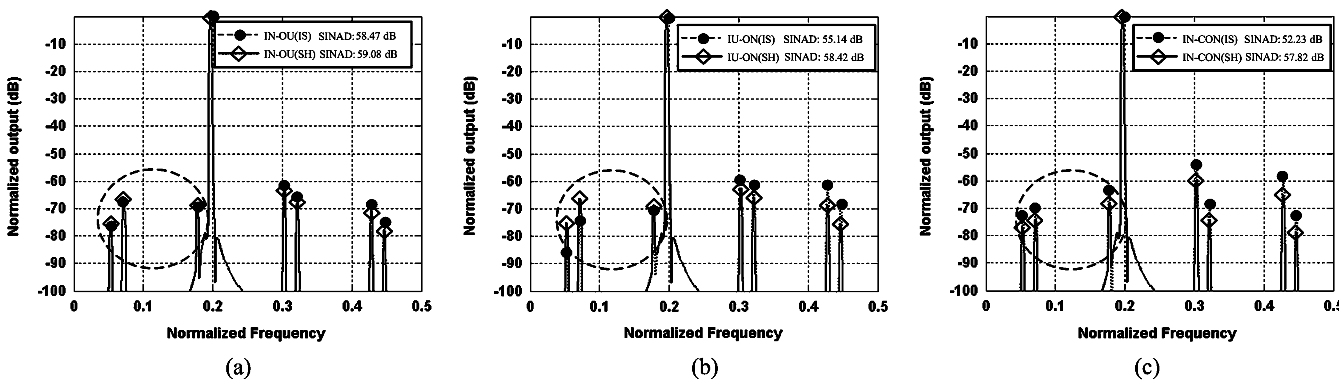

Fig. 2. FFT spectra of output sinusoid for (a) IN-OU, (b) IU-ON and (c) IN-CON processes with both IS and SH output ($a = 0.2, M = 8, \sigma_{r_m} = 0.1\%$).

SINAD and circled parts in the figure for all cases (the sideband magnitudes of SH version of the IN-OU output signal decrease gradually as frequency increases due to the typical uniformly zero-order hold transfer function, while for the latter two cases there are nonuniform modifications for all the sidebands). Therefore, the previous analysis of IU-ON(IS) [11] and IN-CON(IS) [9] cannot be directly applicable for their corresponding SH versions. This paper will present a complete investigation of output signal spectra of practical IU-ON(SH) and IN-CON(SH) processes, including both the closed-form spectra expression as well as their output SINAD. Furthermore, the spurious free dynamic range (SFDR) subjected to in-band image tone will also be derived for a main practical application of the IN-CON(SH) process, i.e., N-path filtering. The corresponding simulation results for demonstration of the accuracy of the derived formula will then also be presented, together with three practical design examples that include a very high-speed data-converter with analog front-end (AFE) filtering [5] and a 21.4 MHz IF sampled-data filter for radio applications, where calculated, simulated and also some measured data from IC implementations will be compared.

II. Signal Spectra With Nonuniformly Holding Output

In this section, the closed-form signal spectra expression with nonuniformly holding output, namely IU-ON(SH) and IN-CON(SH) will be derived. By first obtaining the output signal time-domain description $y(t)$ through a weighted sum of a sample-dependent pulse $h_n(t)$ according to signal's input sampling and output played out timing characteristic, its closed-form spectra expression $Y(\omega)$ will then be obtained by its Fourier transform, and the resulting modulation sidebands in the output spectra will be described by a weighted coefficient function A_k.

A. IU-ON(SH) Systems

Let us first consider the IU-ON(SH) system, which can be described by [9]

$$y_{\mathrm{IU-ON(SH)}}(t)=\sum_{n=-\infty}^{\infty}x(nT)h_n(t-t_n) \qquad (1)$$

with

$$t_n=nT+\Delta_n \qquad (1a)$$

and $T=1/f_s$ is the nominal sampling period and Δ_n is a periodic skew timing sequence with period M (M is usually equal to TI path number N) [9]. For the nonuniform holding output, $h_n(t)$ can be expressed as [9], [10]

$$h_n(t)=u(t)-u(t-T-\Delta_{n+1}+\Delta_n). \qquad (2)$$

Let $n=kM+m$ ($m=0,1,\ldots,M-1$) and timing skew ratio $r_m=\Delta_m/T$, and $\Delta_n=\Delta_{kM+m}=\Delta_m=r_mT$ by the periodicity of Δ_n, then (2) can be written as:

$$h_m(t)=u(t)-u(t-T-r_{m+1}T+r_mT). \qquad (2a)$$

Substituting (1a), (2a) into (1) yields

$$y_{\mathrm{IU-ON(SH)}}(t)=\sum_{m=0}^{M-1}\sum_{k=-\infty}^{\infty}x(kMT+mT)\times h_m(t-kMT-mT-r_mT). \qquad (3)$$

Fig. 3 shows the operations graphically from (1) to (3). Fig. 3(a) shows the uniformly sampled input signal $x(nT)$ and the nonuniformly holding output $y_{\mathrm{IU-ON(SH)}}(t)$, and r_mT are the corresponding amount of timing skew appeared during the playing out instance. Notice that the amount of holding time is equal to $\tau_m=T-r_mT+r_{m+1}T$, and thus the impulse response of the nonuniformly holding function would be a square pulse with pulse width equals to τ_m, as shown in Fig. 3(b). Finally as described in (1), the time domain output waveform $y_{\mathrm{IU-ON(SH)}}(t)$ in Fig. 3(a) can be achieved by multiplication of the input samples $x(nT)$ to the time shifted impulse response (shifted by t_n) as shown in Fig. 3(c), corresponding to the convolution operation.

Applying the Fourier Transform to $y_{\mathrm{IU-ON(SH)}}(t)$ in (3) and after simplification, the output spectrum of the IU-ON(SH) system can be expressed as

$$Y_{\mathrm{IU-ON(SH)}}(\omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}A_{k,\mathrm{IU-ON(SH)}}(\omega)\cdot X\left(\omega-k\frac{2\pi}{MT}\right) \qquad (4)$$

where

$$A_{k,\mathrm{IU-ON(SH)}}(\omega)=\frac{1}{M}\sum_{m=0}^{M-1}\frac{2\sin(\omega(1+r_{m+1}-r_m)T/2)}{\omega}\times e^{-j\omega(1+r_{m+1}-r_m)T/2}e^{-jkm\frac{2\pi}{M}}e^{-j\omega r_mT} \qquad (4a)$$

and $A_{k,\mathrm{IU-ON(SH)}}$ are the weighted terms of the modulation sidebands for the IU-ON(SH) system. Notice that another interesting property of the processes described in (2) to (4a) is that, it provides perfect reconstruction of dc-level signals which is normally not provided by the "ideal" reconstruction method: weighted impulses followed by an ideal low-pass filter. Furthermore, dc constant does not have the error caused by nonunifomly play-out time instance imposed by (2).

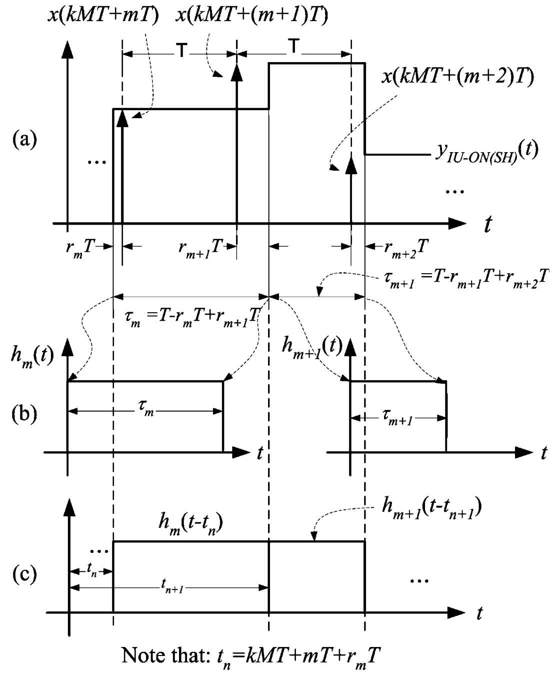

Fig. 3. Operations of IU-ON(SH) process: (a) input sampling sequences and output play out waveforms; (b) impulse response of nonuniformly holding function; (c) time-shifted version of the function in (b).

B. IN-CON(SH) Systems

On the other hand, the output of the IN-CON(SH) system can be described by

$$y_{\mathrm{IN-CON(SH)}}(t)=\sum_{n=-\infty}^{\infty}x(t_n)h_n(t-t_n) \qquad (5)$$

we can obtain $Y_{\mathrm{IN-CON(SH)}}(\omega)$ in the similar form of (4) but with different weight terms to the modulated spectrum sideband, i.e.,

$$A_{k,\mathrm{IN-CON(SH)}}(\omega)=\frac{1}{M}\sum_{m=0}^{M-1}\frac{2\sin(\omega(1+r_{m+1}-r_m)T/2)}{\omega}\times e^{-j\omega(1+r_{m+1}-r_m)T/2}e^{-jkr_m\frac{2\pi}{M}}e^{-jkm\frac{2\pi}{M}}. \qquad (6)$$

The (4) plus (4a) and (6) fully characterize the output signal spectra of the IU-ON and IN-CON processes with output nonuniformly holding effects.

III. Closed-Form Expression for SINAD and SFDR (in IN-CON Systems)

From the derived sidebands weighted terms in (4a) and (6), the modulation sidebands are centered at frequencies of $\omega=\pm\omega_0+k(2\pi)/(MT)$, and thus the signal and image distortion components can be both represented by A_k's with $k=0$ and

$k \neq 0$, respectively. Consequently, the expected signal and distortion power of the output can be evaluated separately by statistical means so as to obtain the SINAD. Similarly, the in-band SFDR for applications of IN-CON(SH) systems, e.g., N -path narrow-band filtering, can be accomplished with the evaluation of A_k with the value of k corresponding to in-band image tone.

A. IU-ON(SH) Systems

Consider first the IU-ON(SH) system described in (4a). For a real input sinusoidal signal with frequency $\omega_0 = 2\pi f_0$, the sidebands are centered at $\omega = \pm\omega_0 + k(2\pi)/(MT)$, with $k = 0$ representing signal and $k = +1, +2, \ldots$ or $k = -1, -2, \ldots$ representing images components at positive and negative frequency axis, respectively. Then, it would be possible to evaluate the sideband components at only $\omega = \omega_0 + k(2\pi)/(MT)$ over the range $[-f_s/2, f_s/2]$ to find the SINAD (with respect to signal only centered at ω_0) over the range of $[0, f_s/2]$, since the images at positive frequencies of $\omega = -\omega_0 + k(2\pi)/(MT)$ with $k = +1, +2, \ldots$ will be directly reflected by the images at negative frequencies of $\omega = \omega_0 + k(2\pi)/(MT)$ with $k = -1, -2, \ldots$, by the symmetry property of fourier transform of real signal. In the following formula derivation, it is assumed that $\omega_0 \neq k(2\pi)/(MT)$, meaning that the signal (and also the sidebands) is not exactly located at integer multiple of f_s/M. On the other hand, simplifying (4a) using $\omega = \omega_0 + k(2\pi)/(MT)$ and considering that r_m and are periodic with period $m = M$, will lead to

$$A_{k,\mathrm{IU-ON(SH)}}\left(\omega_0 + k\frac{2\pi}{MT}\right) = \frac{2\sin(\omega_0 T/2)}{\left(\omega_0 + k\frac{2\pi}{MT}\right)M} \times e^{-j\omega_0 T/2}\sum_{m=0}^{M-1} e^{-j\omega_0 r_m T} e^{-jkr_m\frac{2\pi}{M}} e^{-jkm\frac{2\pi}{M}} \quad (7)$$

and the SINAD can be found by the following [11]:

$$\mathrm{SINAD}_{\mathrm{IU-ON(SH)}} = 10\log_{10}\left[\frac{\left|A_{0,\mathrm{IU-ON(SH)}}(\omega_o)\right|^2}{\sum_k \left|A_{k,\mathrm{IU-ON(SH)}}\left(\omega_o + k\frac{2\pi}{MT}\right)\right|^2}\right]\ \mathrm{dB} \quad (8)$$

where the value of k in the summation is taken in such a way $-\pi f_S \leq \omega_0 + k(2\pi)/(MT) \leq \pi f_S$ and $k \neq 0$. Assume now r_m $(m = 0, 1, 2, \ldots, M-1)$ to be M independent identically distributed (i.i.d.) random variables with Gaussian distribution of zero mean and standard deviation σ_{r_m} $(=\sigma_t/T$ where σ_t is the standard deviation of timing jitter in second). Thus, the expected values of the signal component $|A_{0,\mathrm{IU-ON(SH)}}(\omega_o)|^2$ can be evaluated by substituting $k = 0$ into (7) and multiplying (7) by its complex conjugate as follows:

$$E\left[\left|A_{0,\mathrm{IU-ON(SH)}}(\omega_0)\right|^2\right] = \frac{4}{\omega_0^2 M^2}\sin^2(\omega_0 T/2)\sum_{n=0}^{M-1}\sum_{m=0}^{M-1} E\left[e^{-j\omega_0(r_m - r_n)T}\right]. \quad (9)$$

When $\omega_0 r_m T \ll 1$ (or equivalently $\omega_0 \sigma_{\mathrm{rm}} T \ll 1$) for small values of r_m, (9) can be simplified to

$$\begin{aligned} E\left[\left|A_{0,\mathrm{IU-ON(SH)}}(\omega_0)\right|^2\right] &\approx \frac{4}{\omega_0^2 M^2}\sin^2(\omega_0 T/2) \\ &\quad \times \sum_{n=0}^{M-1}\sum_{m=0}^{M-1} E[1 - j\omega_0(r_m - r_n)T] \\ &\approx \frac{4}{\omega_0^2}\sin^2(\omega_0 T/2). \end{aligned} \quad (10)$$

Similarly, the sideband components $|A_{k,\mathrm{IU-ON(SH)}}(\omega_o + k(2\pi)/(MT))|^2$ can be calculated from (7) as follows:

$$E\left[\left|A_{k,\mathrm{IU-ON(SH)}}\left(\omega_0 + k\cdot\frac{2\pi}{MT}\right)\right|^2\right] \approx \frac{4\sigma_{r_m}^2 T^2}{M}\sin^2(\omega_0 T/2). \quad (11)$$

From (11) it is clear that the expected value of various distortion components in IU-ON(SH) systems is identical for different values of k, thus the total distortion power can be obtained by simply multiplying (11) by $M - 1$. Finally using (10) and (11) in (8), the SINAD of IU-ON(SH) systems can be obtained as follows [19], [21]:

$$\mathrm{SINAD}_{\mathrm{IU-ON(SH)}} = 20\log_{10}\left(\frac{1}{2\pi a\sigma_{r_m}}\right) - 10\log_{10}\left(1 - \frac{1}{M}\right) \quad (12)$$

where $a = f_o/f_s$ is the normalized frequency of the sinusoid. Note that the first term of (12) describes the pure random clock jitter, which is obtained by letting time-skew sequences period $M \rightarrow \infty$. Moreover, the SINAD formula (12) for IU-ON(SH) is interestingly identical to the that for traditional nonuniformly impulse sampling IN-OU(IS) system [7], [9] with small jitter errors. Indeed when $\omega_0\sigma_{r_m}T \ll 1$, the normalized sideband spectra patterns for IN-OU(IS) and IN-OU(SH) are proved to be identical in the Appendix (this is also evident from the spectra and SINAD shown in Fig. 2).

Fig. 4(a) shows the MATLAB FFT simulation results of a IU-ON(SH) system (obtained with $M = 8$) and in Fig. 4(b) the absolute error between the simulated and the calculated results is presented (against a standard deviation σ_{r_m} and normalized signal frequency $a = f_o/f_s$) illustrating the accuracy of the derived formula (12). The error between the IU-ON(SH) output SINAD and that predicted by the formula (12) is well below 0.1 dB (with 10^4 times Monte Carlo simulations), as shown in Fig. 4(b), thus confirming the consistency between the theoretical formula prediction and the FFT simulations results.

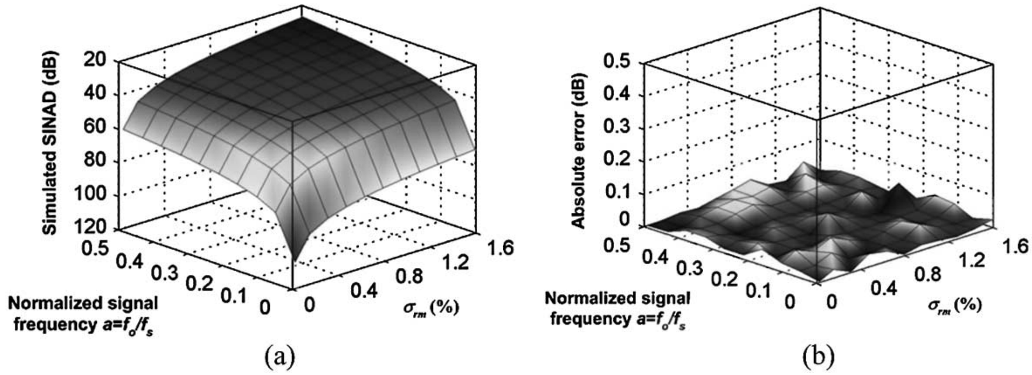

Fig. 4. (a) Simulated SINAD & (b) absolute error between the simulated and calculated SINAD of IU-ON(SH) systems versus normalized frequency a and standard derivation σ_{r_m} by 10^4 times Monte Carlo Simulations ($M = 8$).

B. IN-CON(SH) Systems

For the IN-CON(SH) system, substituting $\omega = \omega_0 + k(2\pi)/(MT)$ in (6) yields

$$\begin{aligned} &A_{k,\mathrm{IN-CON(SH)}}\left(\omega_0 + k\frac{2\pi}{MT}\right) \\ &\quad = \frac{1}{j\left(\omega_0 + k\frac{2\pi}{MT}\right)M}\left\{\sum_{m=0}^{M-1} e^{-jkr_m\frac{2\pi}{M}} e^{-jkm\frac{2\pi}{M}}\right. \\ &\qquad - e^{-j\omega_0 T}\sum_{m=0}^{M-1} e^{-j\omega_0 r_m T} e^{-jkr_m\frac{2\pi}{M}} \\ &\qquad \left.\times\, e^{j\omega_0 r_{m-1} T} e^{-jkm\frac{2\pi}{M}}\right\} \end{aligned} \tag{13}$$

and then, with an expected value of the signal component identical to (10), the expected value of the distortion components can be expressed as follows:

$$\begin{aligned} &E\left[\left|A_{k,\mathrm{IN-CON(SH)}}\left(\omega_0 + k\frac{2\pi}{MT}\right)\right|^2\right] \\ &\quad \approx \frac{2\sigma_{r_m}^2 T^2}{M}\left\{2\sin^2(\omega_0 T/2) - 2\omega_0 T\sin^2(\omega_0 T/2)\right. \\ &\qquad \cdot \frac{1-\cos\left(\omega_0 T + k\frac{2\pi}{M}\right)}{\omega_0 T + k\frac{2\pi}{M}} + \omega_0^2 T^2 \\ &\qquad \cdot \frac{1-\cos\left(\omega_0 T + k\frac{2\pi}{M}\right)}{\left(\omega_0 T + k\frac{2\pi}{M}\right)^2} - \omega_0 T\sin(\omega_0 T) \\ &\qquad \left.\cdot \frac{\sin\left(\omega_0 T + k\frac{2\pi}{M}\right)}{\omega_0 T + k\frac{2\pi}{M}}\right\}. \end{aligned} \tag{14}$$

Unlike in the case of IU-ON(SH), the expected distortion components of the IN-CON(SH) system depends on the value of k, thus it is important to determine the expected values of the total power of all distortion components at $\omega = \pm\omega_0 + k(2\pi)/(MT)$ in the range of $[0, f_s/2]$, and the previous sum in (8) should be modified to include also the images at $\omega = -\omega_0 + k(2\pi)/(MT)$, with k satisfying the following inequality:

$$0 < \pm\omega_0 T + k\frac{2\pi}{M} < \pi \quad \text{and} \quad k \neq 0 \tag{15}$$

The direct calculation of the distortion power sum over this range is not realistic due to the high level of complexity in (14). However, an important characteristic in (14) is that the expected value of the distortion components is a function of $\omega_0 T + k(2\pi)/M$, so for the case of purely random timing jitter ($M \to \infty$), the sum tends to an integral and it is possible to calculate the sum in (8) by integrating (14) over the limit specified in (15). Thus, after the integration, the total distortion power at $\omega = \pm\omega_0 + k(2\pi)/(MT)$ will be given by

$$\begin{aligned} &\sum_k E\left[\left|A_{k,\mathrm{IN-CON(SH)}}\left(\pm\omega_o + k\frac{2\pi}{MT}\right)\right|^2\right] \\ &\quad = 4\sigma_{r_m}^2 T^2\sin^2(\pi a)\cdot\left[0.5 \mp 1.65a\right. \\ &\qquad \left. - \frac{1.85a}{\tan(\pi a)} + \frac{3.82a^2}{\sin^2(\pi a)}\right]. \end{aligned} \tag{16}$$

Then, by using (8), (10), and (16), the closed-form expression of SINAD for IN-CON(SH) system is obtained by [19]–[21]

$$\begin{aligned} \mathrm{SINAD}_{\mathrm{IN-CON(SH)}} &\approx 20\log_{10}\left(\frac{1}{2\pi a\sigma_{r_m}}\right) \\ &\quad - 10\log_{10}\left[1 - \frac{3.7a}{\tan(\pi a)} + \frac{7.64a^2}{\sin^2(\pi a)}\right]. \end{aligned} \tag{17}$$

Fig. 5 is the three-dimension plot showing both the simulated SINAD and the prediction error from the derived formula (17) versus normalized signal frequency a and the standard deviation σ_{r_m} for an IN-CON(SH) system. The absolute error is well below 0.2 dB (with 10^3 times Monte Carlo simulations) showing again the effectiveness of the derived SINAD formula.

Although the SINAD derived in (17) is appropriate for purely random jitter, it can also approximate the SINAD due to periodic timing-skew. Fig. 6 shows the approximation error (in dB) between simulated SINAD and the SINAD calculated using (17) as a function of timing-skew period M, normalized signal frequency a and σ_{r_m}. Fig. 6(a) shows that the error converges to zero as the path and thus the periodic timing-skew periodic M becomes large, and even with low values of M [$M = 2$ in both Fig. 6(a) & (b)], the formula can approximate the SINAD within approximately 2 dB accuracy range (with 10^3 times Monte Carlo simulations) for all range of signal frequencies. Notice that the error grows as the σ_{r_m} becomes

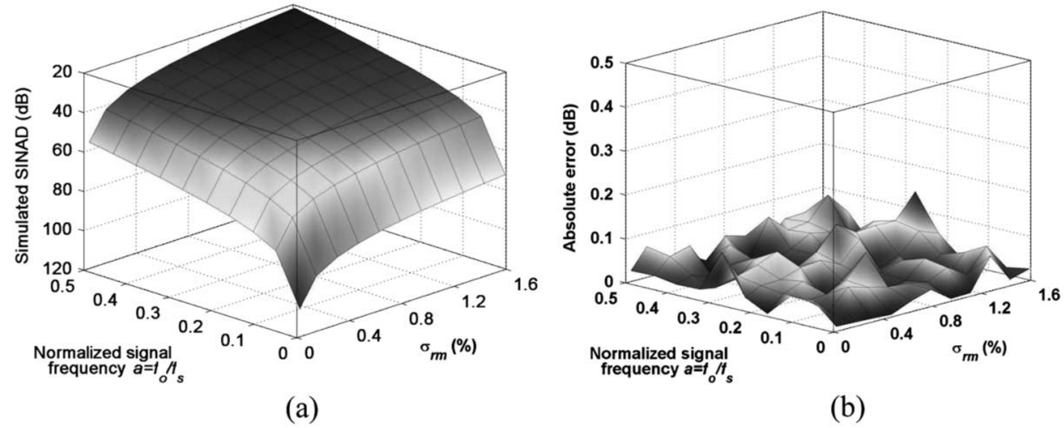

Fig. 5. (a) Simulated SINAD & (b) absolute error between the simulated and calculated SINAD of IN-CON(SH) systems versus normalized frequency a and standard derivation σ_{r_m} by 10^3 times Monte Carlo Simulations.

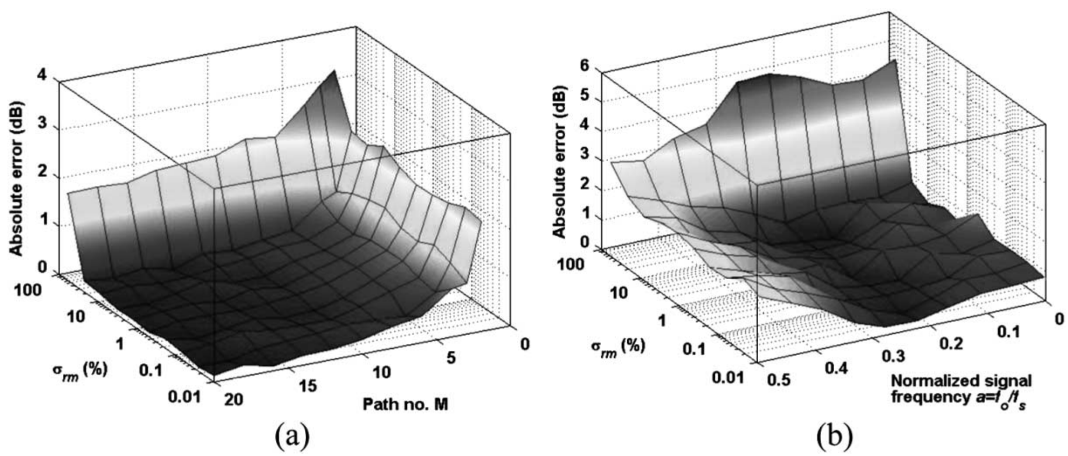

Fig. 6. Absolute error between the simulated and calculated SINAD of IN-CON(SH) systems versus (a) path no. M and standard derivation σ_{r_m} $(a = 0.5)$ and (b) normalized signal frequency a and standard derivation σ_{r_m} $(M = 2)$ by 10^3 times Monte Carlo simulations.

large $(\sigma_{r_m} > 10\%)$, but referred to Fig. 5(a) the corresponding SINAD is already smaller than 20 dB, which proves much less usage for most applications.

In addition to the derived SINAD of the IN-CON(SH) system, it is interesting to investigate also the effects of jitter-induced in-band mirror tone in application of IN-CON(SH) system, e.g., N-path sampled-data filtering which is shown in Fig. 1(c) and is one of the most common methods to construct a narrow-band band-pass filter [3], [16], [17] with passband centered at f_s/N. However, one of the mirror tones caused by periodic timing-skew always appears in-band with signal as shown in Fig. 2(c), thus destroying the performance of narrow-band filtering. For the input signal located at a frequency ω_0, the value of k corresponding to the in-band image can be calculated as follows (which is produced by negative frequency components of the signal):

$$\frac{1}{2}\frac{2\pi}{MT} < -\omega_0 + k\frac{2\pi}{MT} < \frac{3}{2}\frac{2\pi}{MT} \quad \text{or} \quad 0.5 < -Ma + k < 1.5. \tag{18}$$

If the signal is placed close to the center frequency, then $a \approx 1/N = 1/M$ and (18) will give $k = 2$.

The expected value of the mirror tone can be found from (14) with $k = 2$ and $a \approx 1/M$ (using negative frequency components), that is

$$E\left[\left|A_{2,\mathrm{IN-CON(SH)}}\left(-\omega_0 + 2\cdot\frac{2\pi}{MT}\right)\right|^2\right] \approx \frac{16\sigma_{r_m}^2 T^2}{M}\sin^4\frac{\pi}{M} \tag{19}$$

and the expected value of the signal component results from (10) as

$$E\left[\left|A_{0,\mathrm{IN-CON(SH)}}(\omega_0)\right|^2\right] \approx \frac{M^2T^2}{\pi^2}\sin^2\frac{\pi}{M}. \tag{20}$$

Thus, from (19) and (20) the SFDR in the passband subjected to jitter is given by

$$\mathrm{SFDR}_{\mathrm{jitter}} \approx 30\log_{10} M - 20\log_{10}\left(4\pi\sigma_{r_m}\sin\frac{\pi}{M}\right)\ (\mathrm{dBc}). \tag{21}$$

Note that this equation is valid only when $a \approx 1/M$ which is usually true for N-path narrow bandpass filter since the signal is located near the frequency of $f_s/N = f_s/M$. Fig. 7 shows a plot of (21) as function of the timing skew period M and standard

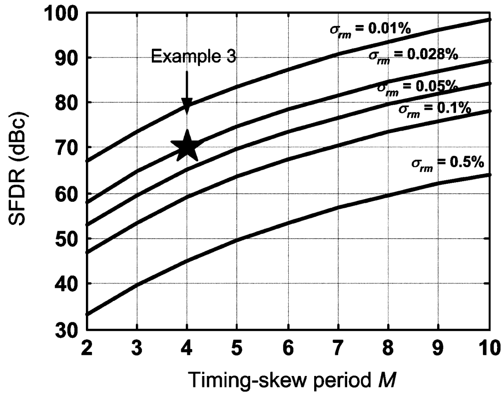

Fig. 7. A plot of variation of in-band SFDR of IN-CON(SH) system versus timing-skew period M and σ_{r_m}.

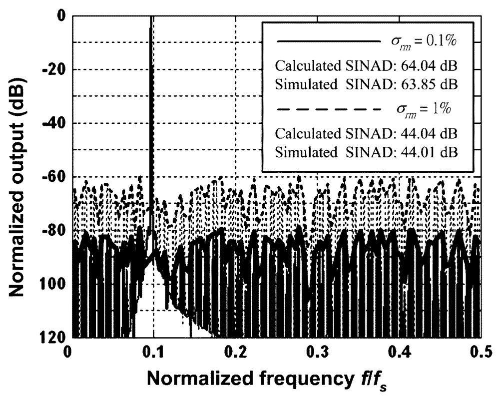

Fig. 8. FFT spectrum of output sinusoid for IU-ON(SH) processes $(a = 0.1, M = 100)$.

deviation σ_{r_m}, and from the figure a considerable reduction on the in-band mirror tone caused by periodic timing-skew is possible via increasing the path number N (and the timing-skew period M).

IV. Practical Applications

To demonstrate the effectiveness of the theories described in the above sections in both purely random timing jitter and periodic timing-mismatch aspects, three practical examples are analyzed as follows:

1) Increased Noise Floor due to Random Jitter Noise in 8 Gs/S Parallel DAC—IU-ON(SH): The random timing jitter is equivalent to a time-skew sequence with period $M \to \infty$. A jittered signal spectrum with $M = 100$, shown in Fig. 8, proofs that the modulated sidebands converge to an increased noise floor as M becomes large. The calculated SINAD for $\sigma_{r_m} = 0.1\%$ and 1% with $a = 0.1$ are 64.04 dB and 44.04 dB, respectively, comparable with the simulated 63.85 dB and 44.01 dB. Moreover, the SINAD increases by 20 dB per decade with respect to the σ_{r_m}, thus showing the consistency between the proposed theoretical prediction and the practical results.

Considering an 8-bit 8 Gsamples/s DAC at the transmission end of a serial-link transceiver as described in [2], which can be classified as an IU-ON(SH) system. To achieve 8-bit accuracy, the SINAD subjected to the noise power caused by the random jitter generated by the on-chip phase lock-loop must be much higher than 50 dB. Thus, at the Nyquist rate of 4 GHz $(a = 0.5)$ the standard deviation σ_{r_m} must be smaller than 0.1%, which is equivalent to a time jitter of 0.125 ps calculated using (12). In current CMOS technology such critical requirement is not easy to be achieved and thus the measured results show that only σ_t of 2.5 ps can be obtained [2].

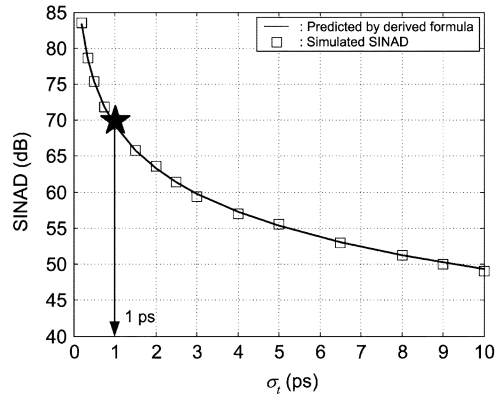

Fig. 9. A plot of comparison between SINAD of example 2 obtained by MATLAB simulation and formula calculation (with $a = 58/320 = 0.18, M = 8$).

2) Modulated Sideband due to Periodic Timing Skew in a 320 MHz Bandpass Multirate Filter—IU-ON/SH): This periodic timing skew will appear from unmatched but fixed propagation delay among the TI phase generation paths. Besides, supply variation caused by dI/dt noise can destroy the matching of the rising edges of the TI phases, thus producing periodic timing skew. Considering as an example of a very high-frequency (output sampling rate at 320 MHz) SC multirate bandpass interpolating filter from [5], [6], the periodic timing-skew mismatch $(M = 8)$ among TI clocks is a dominating factor for the special on-chip multirate phase generator driven by an external master clock. Due to the holding nature at lower input sampling rate of the analog interpolation, the timing-mismatch errors at the input sampling stage are negligible, thus being equivalent to an IU-ON(SH) system. From the derived formula, the standard deviation of the output jitter must be well controlled under the stringent requirement of $\sigma_t < 5$ ps $(\sigma_{r_m} < 5 \text{ ps} \cdot 320 \text{ MHz} = 0.16\%)$ to realize $\text{SINAD} > 60$ dB. Simulation results demonstrate that the worst possible timing-skew can be as large as 100 ps which will completely destroy the system performance without special design and care in the implementation of multirate clock generator [5], [6].

Fig. 9 shows the SINAD from FFT simulated and calculated by derived formula versus σ_t with signal frequency of 58 MHz and $M = 8$. This plot shows that the simulated result tracks very well with the theoretical SINAD curve obtained from (12). The ratio between the measured signal and the timing-jitter noise is around 70 dB in this example [5], [6], corresponding to a fixed periodic timing-skew σ_t of roughly 1 ps, which is well below the requirement of 5 ps. This proves the effectiveness of the various used design and layout techniques [5], [6], and shows also the necessity of a prediction of the allowed timing jitter errors during the design.

3) In-Band Mirror Tone due to Periodic Timing Skew in a 21.4 MHz 4-Path FM Radio IF Filter—IN-

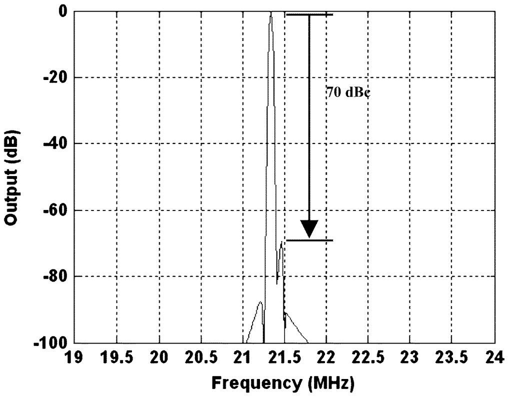

Fig. 10. FFT spectrum of the 21.4 MHz 4-path FM radio IF filter (signal frequency $= 21.34$ MHz, $\sigma_{r_m} = 0.028\%$).

CON(SH): Considering a 21.4 MHz FM radio IF sampled-data filter with bandwidth of 200 kHz implemented using N-path techniques with $N = 4$, and overall sampling frequency $f_s = 85.6$ MHz, it would be equivalent, as described before, to an IN-CON(SH) system. The timing skew among different paths will cause a mirror tone to appear in the passband of the filter. In order to fulfill the requirements of a portable application, the SFDR subjected to in-band mirror tone should be greater than 70 dBc within the band. Using (21) we can calculate the requirement of the σ_{r_m} caused by periodic timing-skew that must be within 0.028%, or equivalently σ_t of 3.3 ps, which corresponds to the star mark shown in Fig. 7. Fig. 10 shows the simulation results corresponding to this example with a signal frequency of 21.34 MHz. The mirror tone appears in-band at 21.46 MHz with a level of 70 dBc, which matches well with the results calculated by the derived formula (21).

V. Conclusion

A complete and exact spectra analysis for jitter-induced nonuniformly holding signals has been presented in this paper, which will be very useful to model accurately the timing-mismatch effects for practical time-interleaved (TI) sampled-data systems. Due to the nonuniform nature of the output SH function in the time-domain, it has been shown that the output signal spectrum is not simply a shaped version (by $\sin(x)/x$) of the corresponding impulse-sampled signal spectrum. The closed-form expressions of both the signal spectra and SINAD for IU-ON(SH) and IN-CON(SH) processes have been also derived in this paper, and they also reveal the fact that the SINAD for IN-OU(IS) is identical to that of IU-ON(SH) with the assumption of $\omega_0\sigma_{r_m}T \ll 1$. Furthermore, the in-band SFDR subjected to skewing mirror tone was also derived for N-path filtering. Finally, the MATLAB simulations have presented the consistency and accuracy of the theoretical analysis results, together with the practical design examples (which include measured data) that illustrate the effectiveness of the derived formula.

Appendix

In the previous sections the SINAD of IU-ON(SH) system is proved to be identical to that of IN-OU(IS) system. Actually from Fig. 2 the normalized sidebands patterns for both systems are the same under the assumption of $\omega_0\sigma_{r_m}T \ll 1$. This Appendix will present the proof of such interesting spectra correlation.

The output spectra of IN-OU(IS) can be represented as follows [9]:

$$Y_{\text{IN-OU(IS)}}(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} A_{k,\text{IN-OU(IS)}}(\omega) \cdot X\left(\omega - k\frac{2\pi}{MT}\right) \quad \text{(A.1)}$$

where

$$A_{k,\text{IN-OU(IS)}}(\omega) = \frac{1}{M}\sum_{m=0}^{M-1}\left(e^{j\omega r_m T}e^{-jkr_m\frac{2\pi}{M}}\right)e^{-jkm\frac{2\pi}{M}} \quad \text{(A.1a)}$$

and substituting $\omega = \omega_0 + k(2\pi)/(MT)$ into (A.1a) yields

$$A_{k,\text{IN-OU(IS)}}\left(\omega_o + k\frac{2\pi}{MT}\right) = \frac{1}{M}\sum_{m=0}^{M-1}\left(e^{j\omega_o r_m T}\right)e^{-jkm\frac{2\pi}{M}} \quad \text{(A.2)}$$

Assuming $\omega_0\sigma_{r_m}T \ll 1$, the magnitude of sideband components can be expressed as: (please see (A.3) and (A.4) at the top of the next page.) For IU-ON(SH) system, the weighted terms of the system are described in (7), which is repeated here for reference

$$A_{k,\text{IU-ON(SH)}}\left(\omega_0 + k\frac{2\pi}{MT}\right) = \frac{2\sin(\omega_0 T/2)}{\left(\omega_0 + k\frac{2\pi}{MT}\right)M}e^{-j\omega_0 T/2} \times \sum_{m=0}^{M-1} e^{-j\omega_0 r_m T}e^{-jkr_m\frac{2\pi}{M}}e^{-jkm\frac{2\pi}{M}}. \quad \text{(A.5)}$$

By expanding the term $e^{-j\omega_0 r_m T}e^{-jkr_m(2\pi/M)}$ of (A.5) into Taylor series $(r_m \ll 1)$ and taking the magnitude, (A.5) yields (please see (A.6) and (A.7) at the top of the next page.) By normalizing the spectra representation (A.6) with respect to (A.7), the magnitude of these normalized sideband components of IU-ON(SH) systems is identical to those of IN-OU(IS) systems described by (A.3), and this completes the proof.

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$$\left|A_{k,\mathrm{IN-OU(IS)}}\left(\omega_o + k\frac{2\pi}{MT}\right)\right| \approx \left|\frac{1}{M}\sum_{m=0}^{M-1}(1+j\omega_o r_m T)e^{-jkm\frac{2\pi}{M}}\right|$$

$$= \begin{cases} \dfrac{\omega_o T}{M}\left|\displaystyle\sum_{m=0}^{M-1} r_m e^{-jkm\frac{2\pi}{M}}\right| & \text{for } k \neq 0, \pm M, \ldots \quad \text{(A.3)} \\ \left|1 + j\omega_o T\left(\dfrac{1}{M}\displaystyle\sum_{m=0}^{M-1} r_m\right)\right| \approx 1 & \text{for } k = 0, \pm M, \ldots \quad \text{(A.4)} \end{cases}$$

$$\left|A_{k\,\mathrm{IU-ON(SH)}}\left(\omega_o + k\frac{2\pi}{MT}\right)\right| = \begin{cases} \dfrac{2T}{M}\sin(\omega_o T/2)\left|\displaystyle\sum_{m=0}^{M-1} r_m e^{-jkm\frac{2\pi}{M}}\right| & \text{for } k \neq 0, \pm M, \ldots \quad \text{(A.6)} \\ \dfrac{2}{\omega_o}\sin(\omega_o T/2) & \text{for } k = 0, \pm M, \ldots \quad \text{(A.7)} \end{cases}$$

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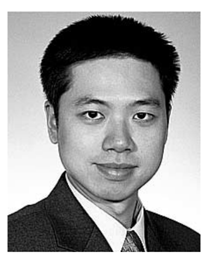

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