

# CCM Operation Analysis and Parameters Design of Negative Output Elementary Luo Converter for Ripple Suppression

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**Abstract**— This paper presents the DC analysis of the Negative Output Elementary Luo Converter (NOELC), which includes the continuous-conduction mode (CCM) voltage gain and the boundary condition between the CCM and the discontinuous-conduction mode (DCM). The main features of the NOELC are the high gain with small ripple and the reverse output. Additionally, we address the parameters design of the NOELC and propose an output voltage ripple estimation method. Through MATLAB Simulink simulation, we further demonstrate that the parameters design and the output voltage estimation method can achieve more accurate results when compared with those from the conventional one.

**Keywords**—Continuous-conduction mode (CCM); Luo converter; parameter design; voltage ripple

## I. INTRODUCTION

PWM dc-dc power converts are widely used in different applications such as renewable energy, computer systems, power supply for portable devices and smart wearables, etc. [1, 2]. Among them, applications in integrated circuit such as audio amplifier, signal generator, double-ended sensors, LCD biasing and light-emitting diode power saving require negative output dc-dc converters [3-8]. A buck-boost converter, for example, can provide inverse output and reach a large gain by varying the duty cycle. However, its output voltage ripple is large and depends on the load current [6]. The larger the load current, the larger the output voltage ripple. The negative output elementary Luo converter (NOELC) can solve the aforementioned problem by adding an output LC filter to the buck-boost converter [9]. Existing papers [9-12] aim to present the control method, its principle and integrations of the Luo converters. Nevertheless, although the output voltage ripple approximation method is presented in [9], it is not accurate and satisfactory as we'll explain here. Hence, we'll propose the parameters design of the NOELC, with a more accurate output voltage ripple estimation method. In Section II, we'll introduce the DC analysis including the continuous-conduction mode (CCM) voltage gain and the boundary between CCM and the discontinuous-conduction mode (DCM). After that, the parameters design of the NOELC will be presented, followed by an output voltage ripple estimation method in Section III. Consequently, the output voltage ripple

can be suppressed by an appropriate design of the parameters. To verify such design and output voltage estimation method, we'll compare the simulation results with the conventional method using MATLAB Simulink in Section IV. Finally, we'll draw the conclusions in Section V.

## II. DC ANALYSIS OF NEGATIVE OUTPUT ELEMENTARY LUO CONVERTER (NOELC)

Fig. 1 shows the circuit configuration of a negative output elementary Luo converter (NOELC), which includes a switch, a diode, two inductors ( $L_1$  and  $L_2$ ) and two capacitors ( $C_1$  and  $C_2$ ), with the load represented by a resistor  $R$ .

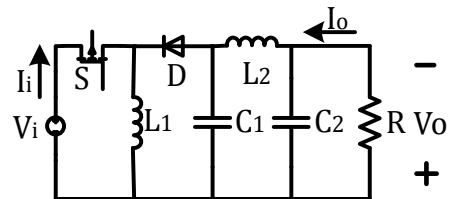


Fig. 1 Negative output elementary Luo converter (NOELC)

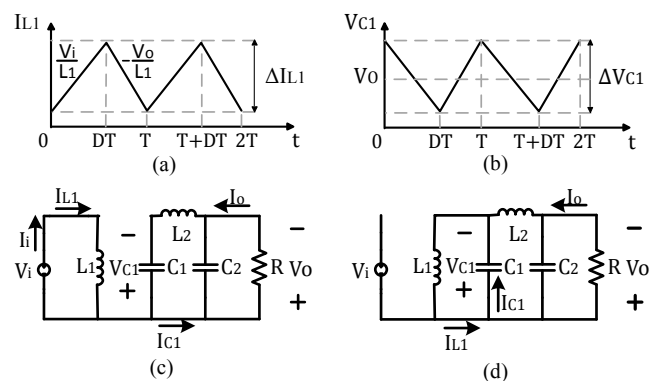


Fig. 2 The operation waveforms and principle of NOELC: (a) inductor current  $I_{L1}$ , (b) capacitor voltage  $V_{C1}$ , (c) equivalent circuit for  $0 < t < DT$ , (d) equivalent circuit for  $DT < t < T$

### A. Continuous-Conduction Mode (CCM) Voltage Gain of Negative Output Elementary Luo Converter

Fig. 2 shows the operation waveforms and principle of the NOELC. In Fig. 2 (a) and (b), when  $0 < t < DT$ , the charge from the source energizes the inductor  $L_1$ . When  $DT < t < T$ , the inductor  $L_1$  releases its energy to charge the capacitor  $C_1$ , hence we get equations (1) and (2) where  $f$  is the switch frequency and  $D$  is the duty cycle of the switch. The inductor current  $i_{L1}$  waveform can be deduced by following (1) and (2) as shown in Fig. 2 (a). In addition, as the energy provided to the load is mainly offered by the capacitor  $C_1$  during  $0 < t < DT$  and offered by inductor  $L_1$  during  $DT < t < T$ . During  $0 < t < DT$ , if the current passes through  $C_2$  is small enough, then since  $I_0 \approx IC_1$ , the capacitor  $C_1$  is driving the load  $R$ . During  $DT < t < T$ , the energy released by the inductor  $L_1$  is stored in the capacitor  $C_1$ , thus, charging it. Hence, the change of capacitor voltage  $\Delta V_{C1}$  can be deduced as in (3) where  $Q_{C1}$  is the charge and the capacitor voltage waveform  $V_{C1}$  is as shown in Fig. 2 (b). From (1) and (2), the transfer function can be obtained as in (4), being the same as the buck-boost converter. In the following,  $I_{L1}$  represents the dc signal, while  $i_{L1}$  represents the ac signal.

$$\Delta i_{L1} = \frac{1}{L_1} \int_0^{DT} V_i dt = \frac{DTV_i}{L_1} = \frac{DV_i}{L_1 f} \quad (1)$$

$$\Delta i_{L1} = \frac{1}{L_1} \int_{DT}^T V_o dt = \frac{(1-D)V_o}{L_1 f} \quad (2)$$

$$\Delta V_{C1} = \frac{\Delta Q_{C1}}{C_1} = \frac{I_o DT}{C_1} = \frac{V_o DT}{RC_1} \quad (3)$$

$$M_V = \frac{V_o}{V_i} = \frac{I_i}{I_o} = \frac{D}{1-D} \quad (4)$$

### B. Boundary between CCM and DCM of the Negative Output Elementary Luo Converter

From Fig. 2(a), if the average inductor current  $I_{L1}$  is smaller than half of its peak-to-peak value, then the converter would enter the DCM. Subsequently, the average of the boundary inductor current  $I_{LB}$  between CCM and DCM can be determined from (5). Then, with (5) and (6), the average of the output boundary load current  $I_{OB}$ , the minimum duty cycle  $D_{min}$  and the boundary load resistance  $R_B$  can be defined as in (7), (8) and (9). Where equations (5) – (9) are the same as in the buck-boost converter. Fig. 3 shows the normalized load current and the resistance boundary between CCM and DCM as a function of  $D$ .

$$I_{L1B} = 0.5 \Delta I_{L1} = \frac{DV_i}{2L_1 f} = \frac{(1-D)V_o}{2L_1 f} \quad (5)$$

$$I_{L1} = \frac{I_i}{D} = \frac{I_o}{1-D} \quad (6)$$

$$I_{OB} = \frac{V_o(1-D_{min})^2}{2fL_1} \quad (7)$$

$$R_B = \frac{V_o}{I_{OB}} = \frac{2fL_1}{(1-D_{min})^2} \quad (8)$$

$$L_{1,min} = \frac{R_{max}(1-D_{min})^2}{2f} \quad (9)$$

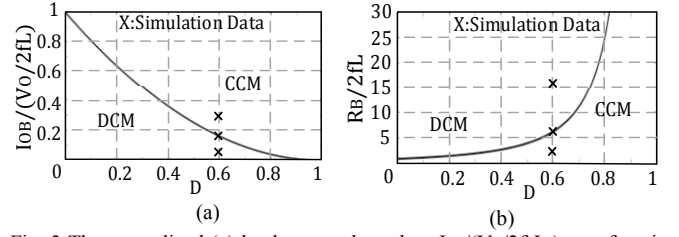


Fig. 3 The normalized (a) load current boundary  $I_{OB}/(V_o/2fL_1)$  as a function of  $D$ , (b) load resistance  $R_B/(2fL_1)$  as a function of  $D$  at the CCM/DCM boundary for the NOELC

## III. PARAMETERS DESIGN OF NEGATIVE OUTPUT ELEMENTARY LUO CONVERTER

Here, we'll present the parameters design ( $L_1$ ,  $L_2$ ,  $C_1$  and  $C_2$ ) of the NOELC. The inductor  $L_1$  can be designed according to the target CCM or DCM operation region (9) as discussed before. The inductor  $L_2$  and capacitor  $C_2$  act as a LC filter, which would be designed to eliminate the high frequency switching signal, for the converter to provide an output voltage with small ripple. The capacitor  $C_1$  aims to hold the charge and keep the capacitor voltage  $V_{C1}$  equal to  $V_o$ .

### A. Design of the Inductor $L_2$ and Capacitor $C_2$

The inductor  $L_2$  and capacitor  $C_2$  act as an LC low pass filter in this converter. The transfer function of the LC filter can be expressed by (10). The cut-off frequency of the LC filter is determined by the  $\omega_0$  in (11) and its gain given by (12),

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\omega_0^2}{s^2 + \omega_0^2} = \frac{\frac{1}{L_2 C_2}}{s^2 + \frac{1}{L_2 C_2}} \quad (10)$$

$$\omega_0 = \sqrt{\frac{1}{L_2 C_2}} \left( \frac{\text{rad}}{s} \right) = \frac{1}{2\pi} \sqrt{\frac{1}{L_2 C_2}} \text{ (Hz)} \quad (11)$$

$$|H(s)| = \left| \frac{1}{1 - L_2 C_2 (2\pi f)^2} \right| \quad (12)$$

### B. Design of the Capacitor $C_1$

Once we have the cut-off frequency of the LC filter, the output voltage ripple will be only determined by the capacitor  $C_1$ . Since the high frequency signal at the output has been filtered out, only the first order harmonic of the switching frequency at the output terminal  $V_o$  exists. To suppress the output voltage ripple, such first order harmonic at the capacitor  $C_1$  should be as small as possible.

The capacitor  $C_1$  voltage ripple function in the time domain can be defined as in (13) with the help of Fig. 2(b). By using a Fourier complex series, we can obtain its harmonic coefficient  $A_k$  in (14) where  $\omega_0 = 2\pi/T$ . By simplifying (14) we can get (15) and determine the first order harmonic coefficient  $A_1$  of the capacitor voltage  $V_{C1}$ . By using the Euler's Equation as in (16), (15) can be expanded as presented in (17). Then the magnitude of  $A_1$  can be expressed as in (18).

$$V_{C1,ripple} = \begin{cases} \frac{V_o D}{RC_1(1-D)} t + \frac{V_o DT}{2RC_1}, & DT - T < t < 0 \\ -\frac{V_o}{RC_1} t + \frac{V_o DT}{2RC_1}, & 0 < t < DT \end{cases} \quad (13)$$

$$A_k = \frac{1}{T} \int_{DT-T}^0 \left( \frac{V_o D}{RC_1(1-D)} t + \frac{V_o DT}{2RC_1} \right) e^{jk\omega_0 t} dt + \frac{1}{T} \int_{DT-T}^0 \left( -\frac{V_o}{RC_1} t + \frac{V_o DT}{2RC_1} \right) e^{jk\omega_0 t} dt \quad (14)$$

$$A_k = \frac{j}{2k\pi} \left[ e^{jk\omega_0 DT} \left( \frac{V_o DT}{2RC_1} - e^{jk\omega_0 T} \left( \frac{V_o(DT-T)}{RC_1(1-D)} + \frac{V_o DT}{2RC_1} \right) \right) + \frac{1}{(2k\pi)^2} \left[ e^{jk\omega_0 DT} \left( -\frac{V_o T}{RC_1} + \frac{V_o DT}{RC_1(1-D)} e^{jk\omega_0 T} \right) - \frac{V_o(1-2D)T}{RC_1} \right] \right] \quad (15)$$

$$e^{j\theta} = \cos(\theta) - j\sin(\theta) \quad (16)$$

$$A_1 = \sin(2\pi D) \left( \frac{1}{2\pi} \frac{V_o DT}{RC_1(1-D)} - \frac{1}{\pi} \frac{V_o D^2 T}{RC_1(1-D)} \right) - \cos(2\pi D) \left[ \frac{1}{4\pi^2} \frac{V_o T}{RC_1(1-D)} - \frac{1}{4\pi^2} \frac{V_o(1-2D)T}{RC_1(1-D)} + j \left[ \frac{1}{\pi} \frac{V_o D^2 T}{RC_1(1-D)} - \frac{1}{4\pi^2} \frac{V_o T}{RC_1(1-D)} \sin(2\pi D) - \frac{1}{2\pi} \frac{V_o DT}{RC_1(1-D)} \cos(2\pi D) \right] \right] \quad (17)$$

$$|A_1| = \sqrt{(\text{Re}\{A_1\})^2 + (\text{Im}\{A_1\})^2} \quad (18)$$

The output voltage ripple can be determined by multiplying the gain of the LC filter (12) and the magnitude of the first order harmonic coefficient of the capacitor  $V_{C1}$  (18), leading to (19).

$$V_{o,ripple} = |A_1| |H(s)| \quad (19)$$

#### IV. SIMULATION RESULTS

In this section, MATLAB Simulink simulations are carried out to verify the parameters design study of the NOELC. The simulation study can be separated into two parts: 1) verification of the CCM/DCM boundary study; and 2) output voltage ripple analysis. In the simulation, the switch and diode loss are neglected for simplicity.

TABLE I.  
THE TEST SIMULATION CONDITIONS OF NOELC FOR CCM/DCM BOUNDARY VERIFICATION

	Case I	Case II	Case III
$f$	1MHz	1MHz	1MHz
$L_1$	1 $\mu$ H	1 $\mu$ H	1 $\mu$ H
$D$	0.6	0.6	0.6
$R$	33.3 $\Omega$	12.5 $\Omega$	7.7 $\Omega$
Normalized load current	0.06	0.16	0.26
Normalized load resistance	16.67	6.25	3.85

TABLE I shows the simulation condition of the NOELC for the CCM/DCM boundary verification. TABLE II and III show

the simulation conditions and the designed parameters for the output voltage ripple analysis of the CCM.

TABLE II.  
THE TEST SIMULATION CONDITIONS FOR OUTPUT VOLTAGE RIPPLE VERIFICATION

	Case IV	Case V
$f$	1MHz	1MHz
$V_i$	1.2V	1.2V
$V_o$	3.3V	3.3V
$I_o$	100mA	150mA
$\Delta V_o$	1~2%	1~2%

TABLE III.  
THE DESIGNED PARAMETERS OF THE NOELC FOR OUTPUT VOLTAGE RIPPLE VERIFICATION

	Case IV & V
$L_1$	1.4 $\mu$ H
$L_2$	0.5 $\mu$ H
$C_1$	0.25 $\mu$ F
$C_2$	0.5 $\mu$ F

#### A. Verification of CCM and DCM Boundary – Cases I, II, III

According to the parameters of the NOELC from TABLE I, Fig. 4 shows the simulated inductor current  $I_{L1}$  for the Cases I, II and III, which verifies the normalized CCM/DCM boundary curve (Fig. 3). In Case I, the normalized load resistance is larger than its boundary value. Then, it enters the DCM region as Fig. 4 shows. In Case II, the normalized load resistance is equal to its boundary value. As a result, it touches the boundary as also shown in Fig. 4; In Case III, the normalized load resistance is smaller than its boundary value. It then enters the CCM region as also illustrated in Fig. 4.

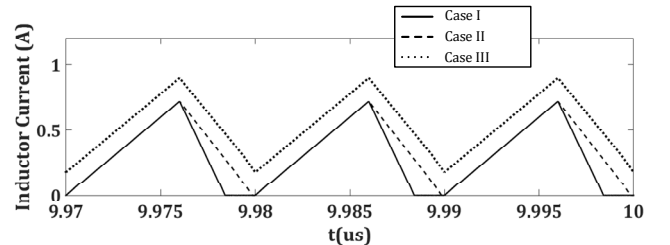


Fig. 4 Inductor Current  $I_{L1}$  of the verification of CCM/DCM boundary study

#### B. Verification of Output Voltage Ripple - Case IV

From TABLE II, the voltage gain is 2.75 and the load resistance is 33 $\Omega$ . Hence  $D$  is 0.734 from (4) and the boundary inductor  $L_{1, \min}$  is 1.17 $\mu$ H from (9). By setting the cut-off frequency  $\omega_0=320$ kHz, we can obtain  $C_2=0.5\mu$ F and  $L_2=0.5\mu$ H from (11). Therefore, the LC filter gain  $|H(s)|$  is 0.113 based on (12). Finally, we substitute the above data into (18) and (19), and the capacitor  $C_1$  can be determined by (20). To achieve 1~2% output voltage ripple, the corresponding capacitor  $C_1$  should be 0.14 $\mu$ F~0.28 $\mu$ F. Therefore,  $C_1=0.25\mu$ F and  $L_1=1.4\mu$ H have been chosen to fulfill such conditions.

$$V_{o,ripple} = \frac{9.39 \times 10^{-9}}{C_1} \quad (20)$$

### C. Verification of Output Voltage Ripple - Case V

From TABLE II, the voltage gain is 2.75 and the load resistance is  $22\Omega$ . Hence,  $D$  is 0.734 from (4) and the boundary inductor  $L_{1,min}$  is  $0.78\mu\text{H}$  from (9). Then we set the cut-off frequency  $\omega_0=320\text{kHz}$ , thus obtaining  $C_2=0.5\mu\text{F}$  and  $L_2=0.5\mu\text{H}$  from (11). Therefore, the LC filter gain  $|H(s)|$  is 0.113 from (12). Finally, we substitute the above data into (18) and (19), and the capacitor  $C_1$  can be determined by (21). To achieve 1~2% output ripple, the capacitor  $C_1$  should be  $0.21\mu\text{F}\sim 0.42\text{nF}$ . Therefore,  $C_1=0.25\mu\text{F}$  and  $L_1=1.4\mu\text{H}$  have been chosen to fulfill such conditions.

$$V_{o,ripple} = \frac{1.40 \times 10^{-9}}{C_1} \quad (21)$$

According to the designed parameters of NOELC as shown in TABLE III, Fig. 5 shows the output voltages for Cases IV and V and TABLE IV summarizes the comparison between this method and the conventional, in which the output voltage ripple (%) is calculated from (22). From TABLE IV, the conventional voltage ripple approximation method [9] is very inaccurate when compared with the proposed estimation method. Simulation results verify the proposed output voltage ripple estimation method, which can help to design the system parameters in order to suppress the voltage ripple. Finally, a summary of the proposed parameters design of the NOELC is shown in TABLE V.

If the total inductance and capacitance for the conventional buck-boost converter are kept the same as the NOELC, that are the total inductance  $L_{buck-boost}=L_1+L_2=0.18\mu\text{H}$  and the total capacitor  $C_{buck-boost}=C_1+C_2=0.75\mu\text{F}$ , the buck-boost converter obtains 3.00% output voltage ripple in Case IV and 4.42% output voltage ripple in Case V, which are about 3 times more than the NOELC. This result also verifies the important value of the NOELC study in this paper.

$$V_{o,ripple}(\%) = \frac{V_{o,ripple}}{V_o} \times 100\% \quad (22)$$

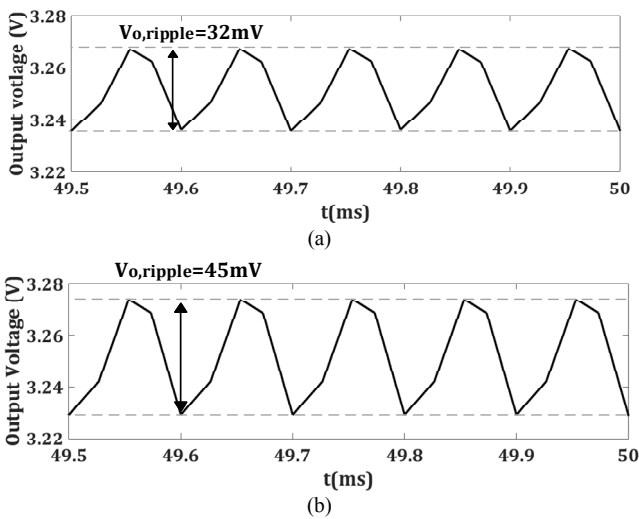


Fig. 5 Output voltage  $V_o$  of (a) Case IV and (b) Case V

TABLE IV.  
COMPARISON OF OUTPUT VOLTAGE RIPPLES BETWEEN THE CONVENTIONAL [9] AND THE PROPOSED METHOD

	Conventional approximation method [9]	Proposed Estimation Method	Actual value
Equation	$\frac{D}{128f^3L_2C_1C_2R}$	$ A_1  H(s) $	/
Voltage Ripple in Case IV (%)	0.084	1.14	0.97
Voltage Ripple in Case V (%)	0.126	1.70	1.35

TABLE V.  
SUMMARY OF PARAMETERS DESIGN OF THE NOELC

Parameter	Equation
$L_1$	$L_{1,min} = \frac{R_{max}(1-D_{min})^2}{2f}$
$L_2$	$ H(s)  = \left  \frac{1}{1-L_2C_2(2\pi f)^2} \right $
$C_1$	$V_{o,ripple} =  A_1  H(s) $
$C_2$	$ H(s)  = \left  \frac{1}{1-L_2C_2(2\pi f)^2} \right $

## V. CONCLUSIONS

This paper presented the CCM operation analysis of a negative output elementary Luo converter (NOELC), which included the CCM voltage gain and the CCM/DCM operation boundary. The parameters design criteria for the NOELC was also determined, as well as the output voltage ripple estimation method which can help to design the system's parameters in order to suppress the voltage ripple. Simulation results were provided to verify the parameters design and the output voltage estimation method in comparison with the conventional method.

## VI. ACKNOWLEDGMENT

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